



GUJARAT TECHNOLOGICAL UNIVERSITY

(Established by Government of Gujarat under Gujarat Act No. : 20 of 2007)

ગુજરાત ટેકનોલોજીકલ યુનિવર્સિટી

(ગુજરાત સરકારના ગુજરાત અધિનિયમ ક્રમાંક : ૨૦/૨૦૦૭ દ્વારા સ્થાપિત)

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Abstract of the Thesis



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Faculty: Mathematics

Discipline/Branch: Science_Mathematics

Title of the Thesis: Differential Geometry of Some Special Finsler Spaces

Abstract

Finsler geometry generalizes Riemannian geometry by permitting the metric tensor to depend on both position and direction. Although foundational work has been established, several important issues concerning special Finsler spaces endowed with particular (α, β) -metrics—such as Berwald-infinite series, Berwald-Matsumoto, generalized Matsumoto, and special cubic metrics—remain unresolved. Notably, there are gaps in their classification into Berwald, Douglas, and weakly Berwald types, as well as in the analysis of their projective transformations and their behavior under Randers-type changes.

The study applies analytical methods from differential geometry and tensor calculus, focusing on essential Finslerian quantities such as the fundamental metric tensor, Cartan tensor, spray coefficients, and curvature tensors, including Berwald and Douglas tensors. It derives necessary and sufficient conditions for projective and dual flatness through rigorous analysis. Additionally, the geodesic equations are formulated and examined within two-dimensional Finsler manifolds and classical geometric surfaces, including cylinders, spheres, cones, catenoids, and pseudo-spheres.

This research establishes necessary and sufficient conditions for Finsler spaces endowed with Berwald-infinite series, Berwald-Matsumoto, and generalized Matsumoto metrics to be classified as Berwald, Douglas, or weakly Berwald spaces. It is demonstrated that any Landsberg space equipped with a special cubic (α, β) -metric is necessarily a Berwald space. Moreover, the study proves that Matsumoto and Kropina metrics possess identical Douglas tensors if and only if both metrics are of Douglas type. Explicit geodesic differential equations are derived for two-dimensional Finsler manifolds under specialized (α, β) -metrics, facilitating the analysis of their geometric behavior on surfaces of revolution. Finally, conditions for the existence of reversible geodesics are established, accompanied by an investigation into the influence of the T-tensor within Randers-changed metric structures.

These findings establish a comprehensive theoretical framework that advances the study of differential geometry, non-Riemannian manifold theory, and associated fields in theoretical physics, particularly in contexts where metric structures depend on direction and generalized geometric spaces are utilized.



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This thesis will help in providing researchers, mathematicians, and theoretical physicists specializing in differential geometry and non-Riemannian manifold theory with explicit formulations of fundamental metric tensors, Cartan tensors, spray coefficients, and curvature tensors. It offers systematic analytical frameworks for investigating specialized (α, β) -metrics and delivers detailed classifications of Berwald, Douglas, and weakly Berwald spaces. Additionally, the derived geodesic differential equations and projective transformation conditions serve as practical references for analyzing complex trajectory behaviors, anisotropic structures, and geometric properties across various surfaces of revolution.

List of Publications:

- 1) S. Prajapati and B. K. Tripathi, *Weakly Berwald Spaces with Generalized Matsumoto Metrics*, Arabian Journal of Mathematics, (2025). **(Scopus, Web of Science | Springer)**.
<https://link.springer.com/article/10.1007/s40065-025-00564-6>
- 2) B. K. Tripathi and S. Prajapati, *Projective Transformation Between Generalized Matsumoto Metric and Kropina Metric with Some Curvature Properties*, Journal of Mathematical Sciences, (2025). **(Scopus | Springer)**. <https://link.springer.com/article/10.1007/s10958-025-07831-z>
- 3) B. K. Tripathi, V. K. Chaubey and S. Prajapati, *The Geodesic Equations in a Two-Dimensional Finsler Manifold Equipped with a Matsumoto-Type Metric-II*, Armenian Journal of Mathematics, **17**(12) (2025), 1-24. **(Scopus, Web of Science)**.
<https://armjmath.sci.am/index.php/ajm/article/view/1376>
- 4) B. K. Tripathi and S. Prajapati, *Berwald and Douglas Spaces of a Finsler Space with a Special Deformed Berwald-Infinite Series Metric*, Journal of Algebra and Related Topics, **13**(2) (2025), 43-52. **(Scopus)**. https://jart.guilan.ac.ir/article_8200.html
- 5) B. K. Tripathi and S. Prajapati, *On Berwald-Matsumoto type Finsler Metrics*, Journal of Finsler Geometry and its applications, **6**(2) (2025), 71-81. **(Scopus)**.
https://jfga.uma.ac.ir/article_3971.html
- 6) B. K. Tripathi, V. K. Chaubey and S. Prajapati, *The Geodesic Equations in a Two-Dimensional Finsler Manifold Endowed with a Matsumoto-Type Metric*, Bulletin of the Institute of Mathematics Academia Sinica (New Series), **20**(3) (2025), 169-195. **(Web of Science)**.
<https://www.math.sinica.edu.tw/bulletin/journals/4926>
- 7) S. Prajapati and B. K. Tripathi, *Equations of Geodesics in Two-dimensional Finsler Manifold with a Special Cubic (α, β) -Metric*, Applications and Applied Mathematics: An International Journal (AAM), **21**(3)(4) (2026). **(Web of Science)**.
<https://digitalcommons.pvamu.edu/aam/vol21/iss3/4/>